Bayesian inversion for properties of the excavation damage zone

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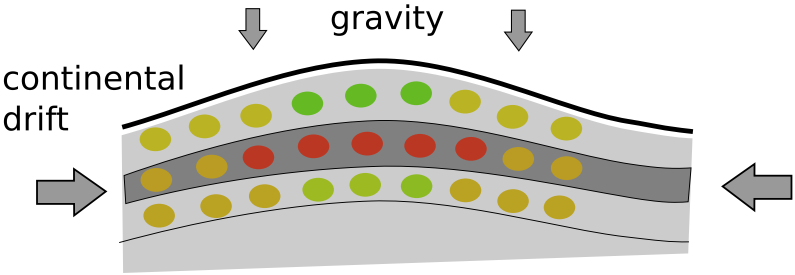
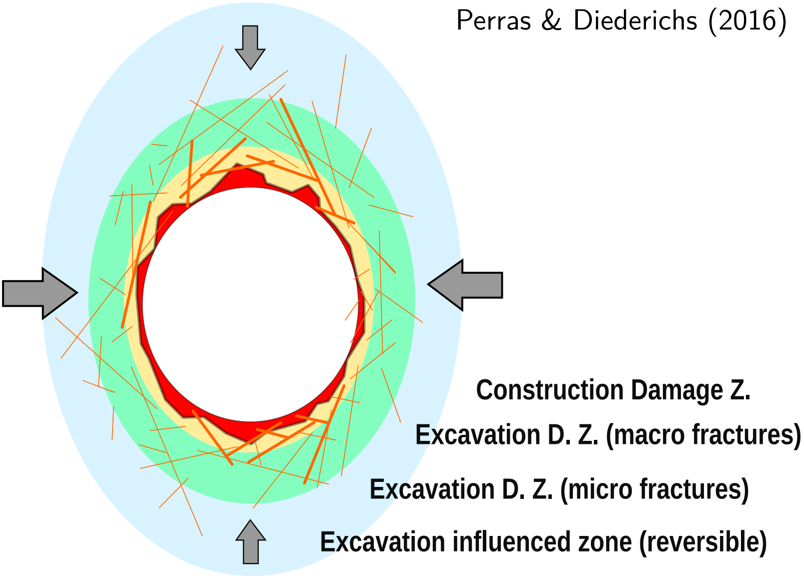
Outline

Initial Stress

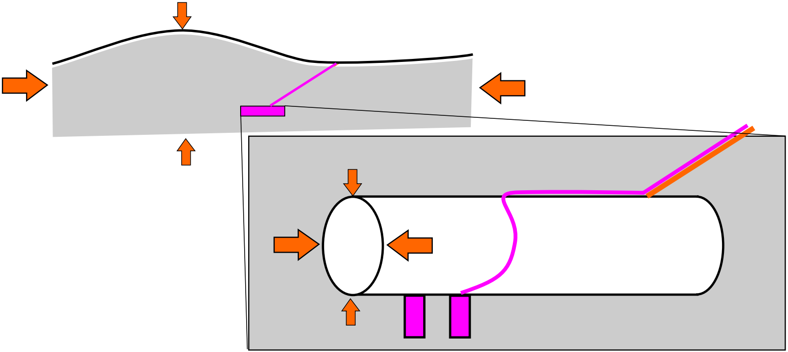


Formation of EDZ

Perras & Diederichs (2016)



Engineering goal: prediction of a safety indicator of EDZ

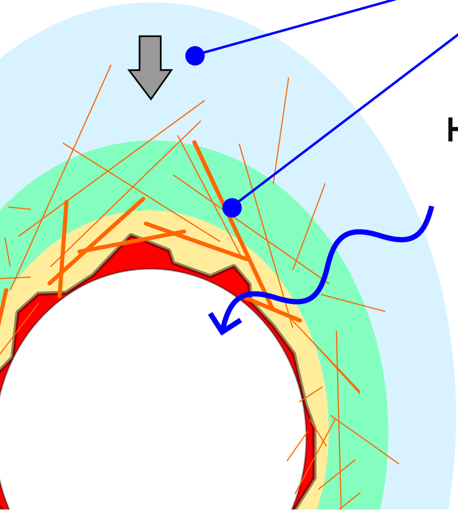


Outline

- Hydro-mechanical model of EDZ formation and development
- Homogenization of fracture networks
- Bayesian inversion, stochastic parameters
- Transport mode and safety indicator

Outline

EDZ processes



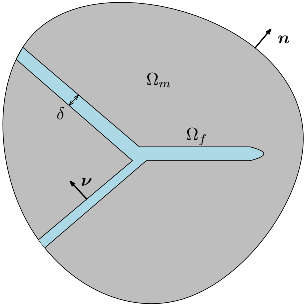
Mechanics:
changes in the stress field
fracture closing / openning
new fractures
plasticity, damage

Hydraulic:
Darcy flow, consolidation
induced permeability changes

Thermal expansion (decay)

Bentonite interactions

HM model: Biot's poroelasticity



Balance / conservation laws

forces: $-\operatorname{div}\boldsymbol{\sigma} + \nabla(\alpha p) = \boldsymbol{f}$

mass: $\partial_t\left(Sp + \operatorname{div}(\alpha\boldsymbol{u})\right) - \operatorname{div}\boldsymbol{q} = g$

Darcy & Hook: $\boldsymbol{q} = -\boldsymbol{K}\nabla p \qquad \boldsymbol{\sigma} = \boldsymbol{C}\nabla\boldsymbol{u}$

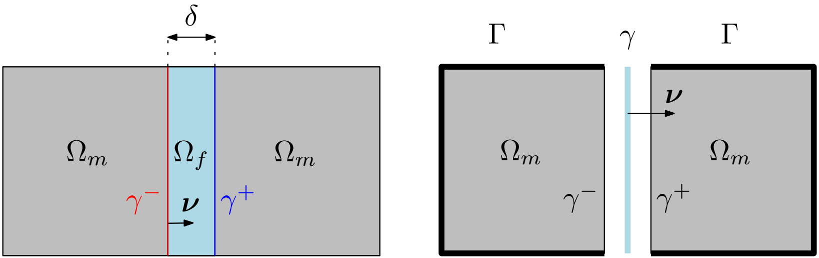
$p, \boldsymbol{q}\cdot\boldsymbol{\nu}, \boldsymbol{u}_\tau, (\boldsymbol{\sigma}\boldsymbol{\nu} - \alpha p\boldsymbol{\nu})_\tau$ continuous on $\partial\Omega_m \cap \partial\Omega_f$

minimal aperture contraint: $\delta + [\![\boldsymbol{u}]\!]\cdot\boldsymbol{\nu} \geq \delta_{min}$

Assumptions:

- $\alpha \geq 0$, $S > 0$, $\boldsymbol{C}$, $\boldsymbol{K}$ piece-wise constant in Ω_m, Ω_f;
- $\boldsymbol{C}$ and $\boldsymbol{K}$ usual symmetries, positive definite

CF model - Geometry assumptions



Assumptions:

- Ω bounded domain in R^d, $d \in \{2, 3\}$, with Lipschitz boundary
- Two subdomains:

$$\Omega_f \text{ and } \Omega_m := \Omega \setminus \overline{\Omega}_f$$

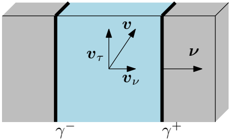
- Straight fracture:

$$\Omega_f = \{\boldsymbol{x} + s\boldsymbol{\nu};\ \boldsymbol{x} \in \gamma,\ s \in (-\tfrac{\delta}{2}, \tfrac{\delta}{2})\}$$

Normal and tangential calculus

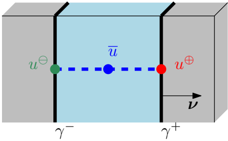
Decomposition into normal and tangential part:

$$\boldsymbol{v} = \boldsymbol{v}_\nu + \boldsymbol{v}_\tau$$
$$\nabla \boldsymbol{v} = \nabla_\nu \boldsymbol{v} + \nabla_\tau \boldsymbol{v}$$
$$\operatorname{div} \boldsymbol{v} = \operatorname{div}_\nu \boldsymbol{v} + \operatorname{div}_\tau \boldsymbol{v}$$



Integral mean over fracture:

$$\overline{u} := \frac{1}{\delta}\int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} u \,\mathrm{d}\boldsymbol{\nu}$$



Average and jump operators:

$$\{\!\{u\}\!\} = \tfrac{1}{2}(u^\oplus + u^\ominus) \quad [\![u]\!] = u^\oplus - u^\ominus$$

Approximate gradient on $\gamma^\pm$:

$$\widetilde{\nabla}^\oplus(u,\overline{u}) := \nabla_\tau \overline{u} + \frac{2}{\delta}(u^\oplus - \overline{u})\boldsymbol{\nu}, \quad \widetilde{\nabla}^\ominus(u,\overline{u}) := \nabla_\tau \overline{u} - \frac{2}{\delta}(u^\ominus - \overline{u})\boldsymbol{\nu}$$

CF poroelasticity model

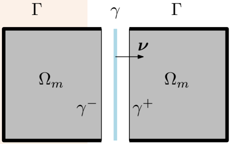
- Approximate normal flux :
  $\boldsymbol{K}\nabla p_{|\Omega_f} \cdot \boldsymbol{\nu} \approx \boldsymbol{K}\widetilde{\nabla}(p, \overline{p}) \cdot \boldsymbol{\nu} =: \varphi^{\oplus}(p, \overline{p})$
- and traction on $\gamma^{\pm}$ $(\boldsymbol{C}\nabla u_{|\Omega_f})\boldsymbol{\nu} \approx (\boldsymbol{C}\widetilde{\nabla}(\boldsymbol{u}, \overline{\boldsymbol{u}}))\boldsymbol{\nu} =: \boldsymbol{t}^{\oplus}(\boldsymbol{u}, \overline{\boldsymbol{u}})$
- New unknowns $P = (P_m, P_f)$, $\boldsymbol{U} = (\boldsymbol{U}_m, \boldsymbol{U}_f)$
- Time $I = (0, T)$, init. c. $P(0, \cdot) = P_0$, b.c. $\boldsymbol{U} = \boldsymbol{0}$, $P = 0$

Continuum-fracture Biot model

$$\left.\begin{aligned} -\operatorname{div}(\mathrm{C}\nabla \boldsymbol{U}_m) + \alpha\nabla P_m &= \boldsymbol{F}_m \\ \partial_t\left(SP_m + \alpha \operatorname{div} \boldsymbol{U}_m\right) - \operatorname{div}(\mathrm{K}\nabla P_m) &= G_m \end{aligned}\right\} \text{ in } I \times \Omega_m$$

$$\left.\begin{aligned} -\delta \operatorname{div}_\tau(\boldsymbol{C}\{\!\{\widetilde{\nabla}\boldsymbol{U}\}\!\}) + \delta\alpha\{\!\{\widetilde{\nabla}P\}\!\} - [\![\boldsymbol{t}(\boldsymbol{U})]\!] &= \boldsymbol{F}_f \\ \delta\partial_t(SP_f + \alpha\{\!\{\widetilde{\operatorname{div}}\boldsymbol{U}\}\!\}) - \delta \operatorname{div}_\tau(\boldsymbol{K}\{\!\{\widetilde{\nabla}P\}\!\}) - [\![\varphi(P)]\!] &= G_f \end{aligned}\right\} \text{ in } I \times \gamma$$

$$\left.\begin{aligned} \mathrm{K}\nabla P_m^{\oplus} \cdot \boldsymbol{\nu} &= \varphi^{\oplus}(P) \\ ((\mathrm{C}\nabla \boldsymbol{U}_m)^{\oplus}\boldsymbol{\nu})_\tau &= \boldsymbol{t}_\tau^{\oplus}(\boldsymbol{U}) \\ \delta + [\![\boldsymbol{U}_m]\!] \cdot \boldsymbol{\nu} &\geq \delta_{min} \end{aligned}\right\} \text{ on } I \times \gamma^{\pm}$$



A priori estimate

In matrix Ω_m:

$$\alpha \int_{\Omega_m} \partial_t(\nabla P_m)\partial_t \boldsymbol{U}_m + \alpha \int_{\Omega_m} \partial_t P_m \partial_t(\operatorname{div} \boldsymbol{U}_m) = 0$$

In fracture γ:

$$\alpha \int_{\gamma} \partial_t([\![P_m]\!] \cdot \boldsymbol{\nu})\partial_t \boldsymbol{U}_f + \alpha \int_{\gamma} \partial_t P_f \partial_t([\![\boldsymbol{U}_m]\!] \cdot \boldsymbol{\nu}) \neq 0$$

Reason: Lack of Green's formula for approximate gradients

$$\int_{\gamma} \{\!\{\tilde{\nabla} P\}\!\} \cdot \boldsymbol{U}_f \neq - \int_{\gamma} P \{\!\{\widetilde{\operatorname{div}} \boldsymbol{U}\}\!\}$$

Workaround: assume $\boldsymbol{K}_f \gg \boldsymbol{K}_m$ in derivation of CF model

$$p^{\oplus} = \overline{p} + O(\delta), \quad \overline{\nabla p} = \nabla_{\tau}\overline{p} + O(\delta)$$

Poroelasticity: Weak formulation

Product function spaces for displacement and pressure:

$$\boldsymbol{\mathcal{V}} := H^1_\Gamma(\Omega_m;\mathrm{R}^d)\times H^1_0(\gamma;\mathrm{R}^d) \qquad \boldsymbol{\mathcal{K}} := \{\boldsymbol{V}\in\boldsymbol{\mathcal{V}};\ \delta+[\![\boldsymbol{V}_m]\!]\cdot\boldsymbol{\nu} \ge \delta_{min}\}$$

$$\mathscr{L} := L^2(\Omega_m)\times L^2(\gamma) \quad \mathcal{V} := H^1_\Gamma(\Omega_m)\times H^1_0(\gamma) \quad \mathcal{X} := L^\infty(I;\mathcal{V})\cap H^1(I;\mathscr{L})$$

Bilinear forms:

$$\begin{aligned}
a(\boldsymbol{U},\boldsymbol{V}) &:= \int_{\Omega_m} \boldsymbol{C}\nabla\boldsymbol{U}_m : \nabla\boldsymbol{V}_m + \delta\int_\gamma \{\!\{\boldsymbol{C}\tilde{\nabla}\boldsymbol{U} : \tilde{\nabla}\boldsymbol{V}\}\!\} \\
b(P,\boldsymbol{V}) &:= \alpha\int_{\Omega_m} P_m \,\mathrm{div}\,\boldsymbol{V}_m + \delta\alpha\int_\gamma P_f\{\!\{\widetilde{\mathrm{div}}\boldsymbol{V}\}\!\} \\
c(P,Q) &:= \int_{\Omega_m} (SQ_m\partial_t P_m + \boldsymbol{K}\nabla P_m\cdot\nabla Q_m) \\
&\quad + \delta\int_\gamma SQ_f\partial_t P_f + \delta\int_\gamma \{\!\{\boldsymbol{K}\tilde{\nabla}P\cdot\tilde{\nabla}Q\}\!\}
\end{aligned}$$

Poroelasticity: Weak formulation

Problem (HM)

Find $\boldsymbol{U} \in H^1(I;\mathcal{K})$ *and* $P \in \mathcal{X}$ *such that* $P(0,\cdot) = P_0$ *and for all* $\boldsymbol{V} \in \mathcal{K}$, $Q \in \mathcal{V}$, *a.a.* $t \in I$:

$$a(\boldsymbol{U}(t), \boldsymbol{V} - \boldsymbol{U}) - b(P(t), \boldsymbol{V} - \boldsymbol{U}) \leq \langle \boldsymbol{F}(t), \boldsymbol{V} - \boldsymbol{U} \rangle_{\boldsymbol{\mathcal{V}}}$$

$$b(Q, \partial_t \boldsymbol{U}(t)) + c(P(t), Q) = \langle G(t), Q \rangle_{\mathscr{L}}$$

Poroelasticity: Fixed-stress splitting

Stabilization form:

$$c_\beta(P,Q) := \int_{\Omega_m} \beta Q_m \partial_t P_m + \int_\gamma \beta Q_f \partial_t P_f, \ \beta > 0$$

Problem (HM-FS)

Given P^{n-1}, find $(\boldsymbol{U}^n, P^n) \in H^1(I;\mathcal{K}) \times \mathcal{X}$ such that

$$a(\boldsymbol{U}^n(t), \boldsymbol{V} - \boldsymbol{U}^n) - b(P^{n-1}(t), \boldsymbol{V} - \boldsymbol{U}^n) \le \langle \boldsymbol{F}(t), \boldsymbol{V} - \boldsymbol{U}^n \rangle_{\boldsymbol{\mathcal{V}}}$$
$$b(Q, \partial_t \boldsymbol{U}^n(t)) + c(P^n(t), Q) + c_\beta(P^n - P^{n-1}, Q) = \langle G(t), Q \rangle_{\mathscr{L}}$$

for all $\boldsymbol{V} \in \mathcal{K}$, $Q \in \mathcal{V}$, a.a. $t \in I$

- Mapping for fixed point argument:

$$\mathcal{M} : \mathcal{X} \to \mathcal{X}, \qquad \mathcal{M} : P^{n-1} \mapsto P^n$$

Poroelasticity: Well-posedness

Theorem

Let $\beta > \frac{\alpha^2}{2(\frac{2\mu}{d}+\lambda)}$. Then $\mathcal{M}$ is a contraction in $\mathcal{X}$. The fixed point of this mapping is the unique solution of the problem (HM). The contraction constant takes the value $1/\omega$, which is smallest when $\beta = \frac{\alpha^2}{2(\frac{2\mu}{d}+\lambda)}$.

Notes about the proof

- Mechanics: Existence of the mapping $P \mapsto \boldsymbol{U}$ from $H^1(I;\mathscr{L})$ to $H^1(I;\mathcal{K})$ proved using theory of variational inequalities and a Korn-type inequality in $\boldsymbol{\mathcal{V}}$.
- Flow: Existence of the mapping $\boldsymbol{U} \mapsto P$ from $H^1(I;\mathcal{K})$ to $\mathcal{X}$ proved using Galerkin method.
- $\mathcal{M}$ is a contraction w.r. to certain metric in $\mathcal{X}$.

References:

Proof is based on the arguments of

A. Mikelić, M.F. Wheeler: Convergence of iterative coupling for coupled flow and geomechanics, Comput. Geosci. 17, 2013.

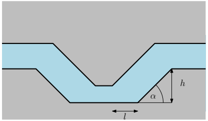
J. W. Both et al. Robust fixed stress splitting for Biot's equations in heterogeneous media. Applied Mathematics Letters, 68:101–108, 2017.

For full proof see:

J. B., J. Stebel: Mixed-dimensional model of poroelasticity, preprint at ResearcherGate.

Generalizations

- Stress dependent matrix permeability
- Cubic law for fracture permeability:
  $$\boldsymbol{K}_{|\Omega_f} = \frac{(\delta + [\![\boldsymbol{U}]\!] \cdot \boldsymbol{\nu})^2}{12} \boldsymbol{I}$$
  small variations of $[\![\boldsymbol{U}]\!] \cdot \boldsymbol{\nu}$ required for convergence
- Contact with shear dilation



$$\delta_{min} = \delta_{min}(\boldsymbol{U})\ldots \text{ bounded, Lipschitz continuous}$$

Solution by successive approximations:
$\mathcal{P} : \delta_{min} \mapsto \boldsymbol{U}; \qquad \boldsymbol{U}^{n+1} := \mathcal{P}(\delta_{min}(\boldsymbol{U}^n))$

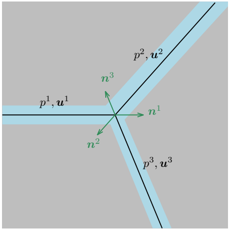
- Coulomb friction on fractures.

Numerical solution: FE spaces

- Generalization to immersed fractures, crossings and branching:

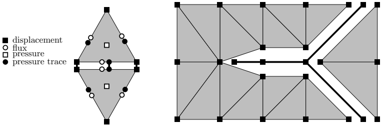
$$\sum_i \boldsymbol{C}_f \{\!\{\widetilde{\nabla} \boldsymbol{u}^i\}\!\} \boldsymbol{n}^i = \boldsymbol{0}$$

$$\sum_i \boldsymbol{K}_f(\boldsymbol{u}^i) \{\!\{\widetilde{\nabla} p^i\}\!\} \cdot \boldsymbol{n}^i = 0$$



- Compatible discretization of domain and fracture
- FE spaces: P1/MH

displacement	P_1
pressure	P_0
flux	RT_0
pressure trace	P_0 on edges



Numerical solution: Discrete problem

- Algebraic form of (HM) after time and space discretization:

$$\min_{(\boldsymbol{U},P)} \left(\frac{1}{2} \begin{bmatrix} \boldsymbol{M} & \boldsymbol{D}^\top \\ \boldsymbol{D} & \boldsymbol{H}(\boldsymbol{U}) \end{bmatrix} \begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{P} \end{bmatrix} - \begin{bmatrix} \boldsymbol{F} \\ \boldsymbol{G} \end{bmatrix} \right) \cdot \begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{P} \end{bmatrix}, \text{ s.t. } \boldsymbol{B}\boldsymbol{U} \le \boldsymbol{d}(\boldsymbol{U})$$

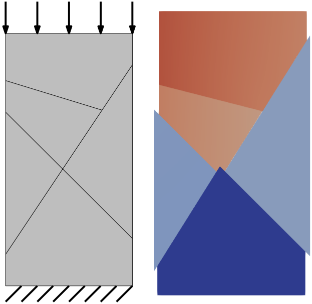
- Fixed-stress splitting:

$$(\boldsymbol{H}(\boldsymbol{U}_{n-1}) + \boldsymbol{H}_\beta)\boldsymbol{P}_n = \boldsymbol{G} + \boldsymbol{H}_\beta \boldsymbol{P}_{n-1} - \boldsymbol{D}\boldsymbol{U}_{n-1}$$

$$\min_{\boldsymbol{U}_n} \left(\frac{1}{2} \boldsymbol{M}\boldsymbol{U}_n \cdot \boldsymbol{U}_n - (\boldsymbol{F} - \boldsymbol{D}^\top \boldsymbol{P}_n) \cdot \boldsymbol{U}_n \right) \text{ s.t. } \boldsymbol{B}\boldsymbol{U}_n \le \boldsymbol{d}(\boldsymbol{U}_{n-1})$$

- Solution by quadratic programming methods (PERMON toolbox/PETSc)
- Implementation: Flow123d (in-house code)

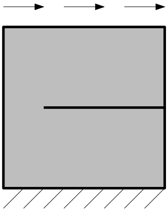
Numerical examples: Contact mechanics



- Young modulus: rock 10^9, fractures 10^4
- fracture stiffness prevents material disintegration
- contact conditions averaged over fracture elements - easy treatment of intersections, does not exclude local penetrations

Numerical examples: Shear dilation

Comparison of contact vs. shear dilation.



$$\delta_{min,0}(t) := 0$$

$$\delta_{min,1}(t) := \begin{cases} 0 & t \le 0.01, \\ t - 0.01 & t \in [0.01, 0.03], \\ 0.02 & t \ge 0.03. \end{cases}$$

Young modulus rock	1
Young modulus fracture	10^{-4}
Poisson's ratio rock	0.25
Poisson's ratio fracture	0.25
Domain dimensions	1×1
Fracture cross-section	0.02

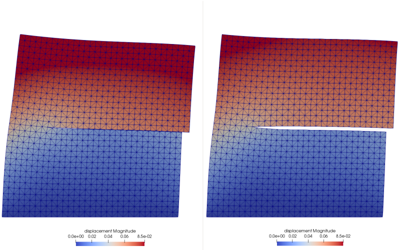


Figure: Left: $\delta_{min,0}$, right: $\delta_{min,1}$.

Outline

Empirical formulas

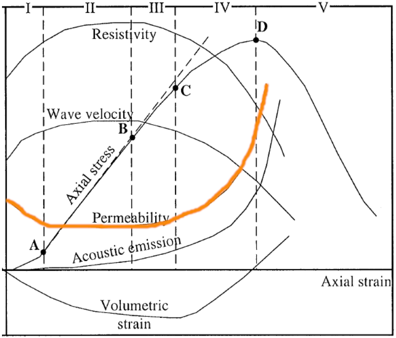
Kožený; Carman (1956):

$$k = C\frac{e^3}{1+e} \approx C\theta^2$$

$e = \theta/(1-\theta)$ - void ratio

Souley et al. (2001),
Rutqvist et al. (2009)

$$k = \left[k_r + \Delta k_{max}\exp(\beta\sigma'_m)\right]\exp(\gamma\Delta\sigma_d)$$



Effective mean stress: $\sigma'_m = \frac{1}{3}(\sigma_1+\sigma_2+\sigma_3) - P$
Deviatoric stress, von-Mises:

$$\sigma_d = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$$

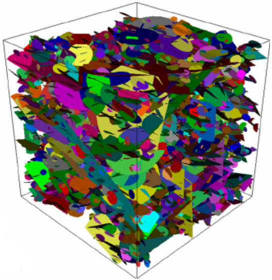
Conceptual model of crystalline rock

- fragile and hard rock $\Rightarrow$ fracture network
- self-similar, scale invariant fracture network
- scale invariance of material properties: permeability, porosity, ...
  Neuman, 2008

Motivation for numerical homogenization:

- interpolation between fracture networks and continuum model
- multilevel Monte-Carlo method for CF models
- physically justified constitutive model for permeability, ...
- scale extrapolation of sample/laboratory measurements

Discrete Fracture Network (DFN)



- **Number of fractures** of size $r \in [\underline{r}, \overline{r}]$ in unite volume:

$$N \sim Poisson(\lambda),\ \lambda = P_{30}[\underline{r}, \overline{r}]$$

- **Fracture size** - power law:

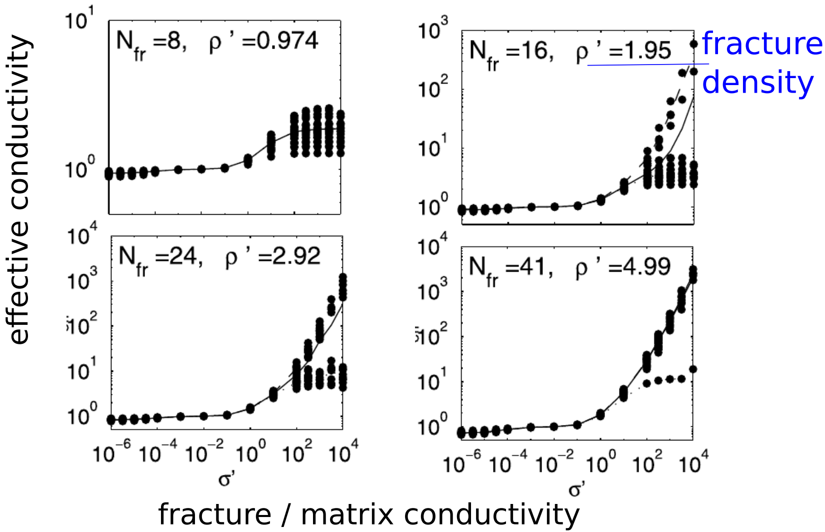
$$f(r) = \frac{1}{f_0} r^{-(\kappa+1)},$$

- **Aperture** correlated to the fracture size.
- **Orientation** - Fischer distribution, anisotropy
- **Position** - uniform, homogeneous

Questions:
Scale invariance, correlations, mechanical energy, ...

Lei et al., 2017

Percolation theory



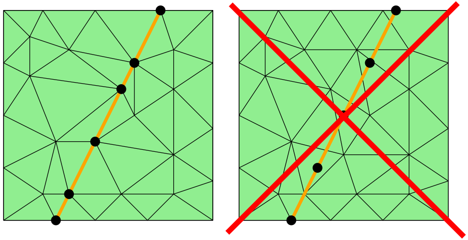
Bogdanov, 2003

Meshing

Regularization of the fracture network

- add fractures from the biggest
- attach to existing points and segments
- drop small segment ends
- try to avoid small angle segment intersections

Conforming 1d-2d mesh (GMSH)



Effective permeability tensor

- Effective tensor (2D):

$$\boldsymbol{K} = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

- prescribe Dirichlet condition:

$$\nabla p_i(x) = \varepsilon(\cos\,\alpha_i, \sin\,\alpha_i)$$

- Volume average of the velocity:

$$\varepsilon \boldsymbol{v}_i = \sum_{E\in\mathcal{T}} \delta_E |E| v_E$$

- Least square fit of the tensor:

$$\begin{bmatrix} \dots & & \\ \cos\alpha_i & \sin\alpha_i & 0 \\ 0 & \cos\alpha_i & \sin\alpha_i \\ \dots & & \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \approx \begin{bmatrix} \dots \\ v_{i,x} \\ v_{i,y} \\ \dots \end{bmatrix}$$

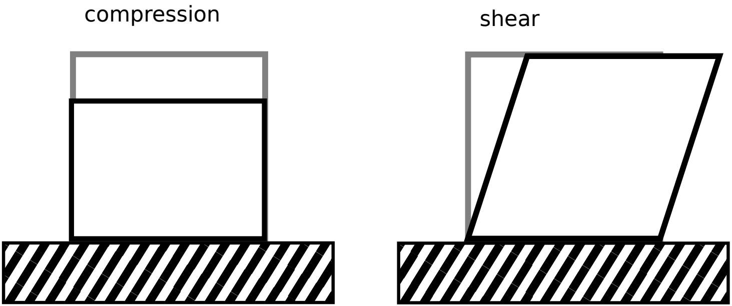
Dependency on the stress

Distinct Element Method, 2D
Min, Rutqvist, ..., 2004 : Stress-dependent permeability . . . ,
Bidgoli et al., 2013

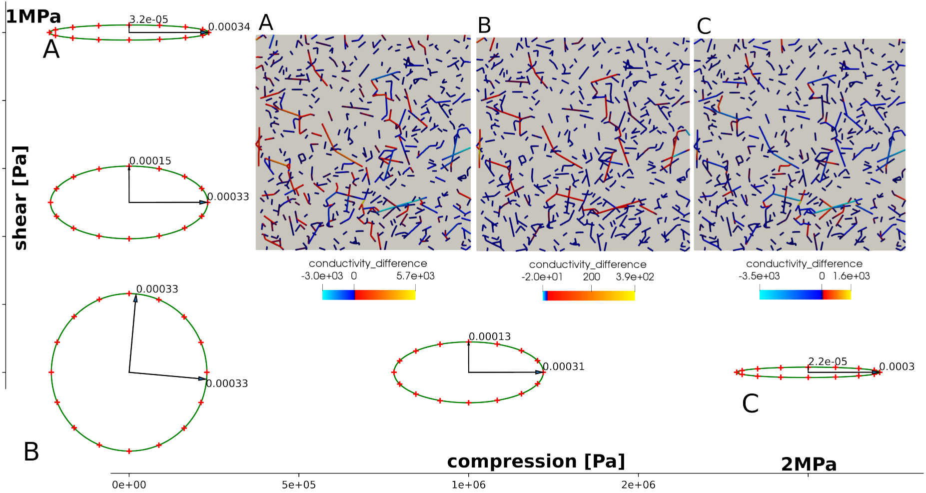
Dependency on the stress

Distinct Element Method, 2D
Min, Rutqvist, ..., 2004 : Stress-dependent permeability . . . ,
Bidgoli et al., 2013

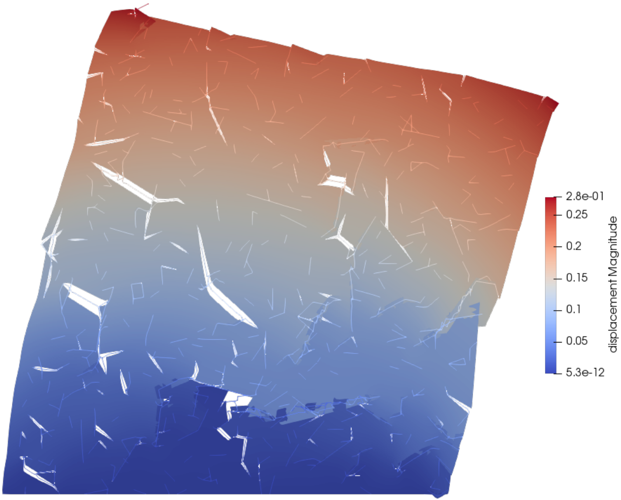
Considering $p \ll \sigma$.



Preliminary results



Detail



Magnification: 400

Future work

fracture networks

- regularization for 3D networks
- correlated positions, energy based distributions

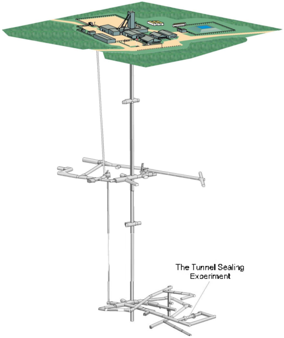
HM homogenization

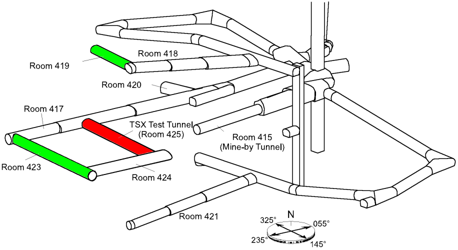
- global shift and rotation constraints
- shear dilation, friction
- homogenization of the mechanical properties
- comparison to empirical formulas

Outline

TSX Experiment

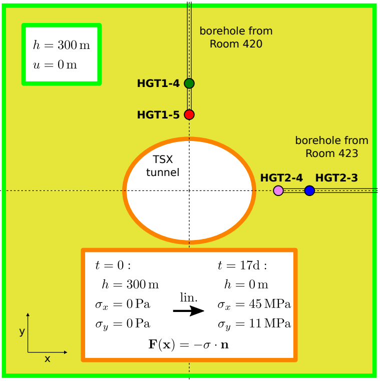
TSX = Tunnel Sealing Experiment,
Canada, Manitoba, Lac du Bonnet, Underground Research Lab.

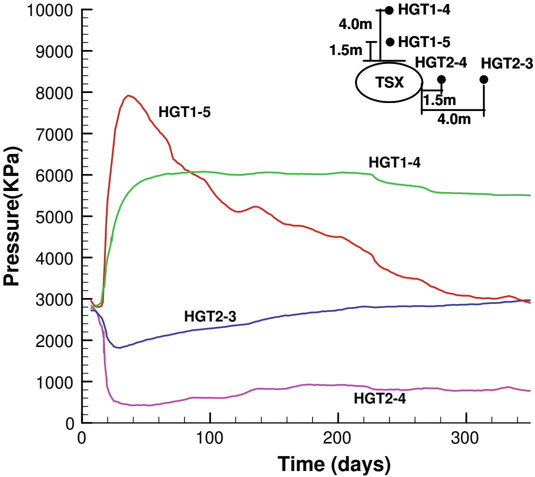




Chandler, 2002

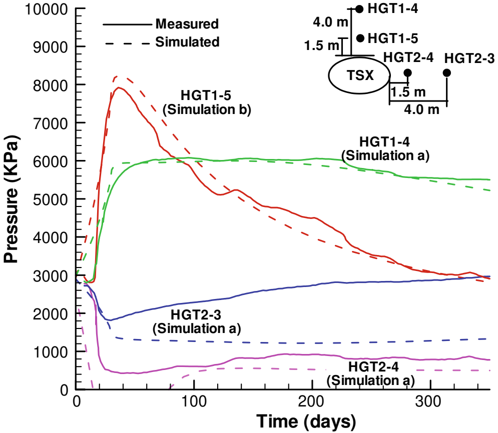
Pore pressure - mine by experiment





Previous work

- plastic mechanical model (Mohr-Coulomb)
- empiric non-linear relation for conductivity near tunnel wall



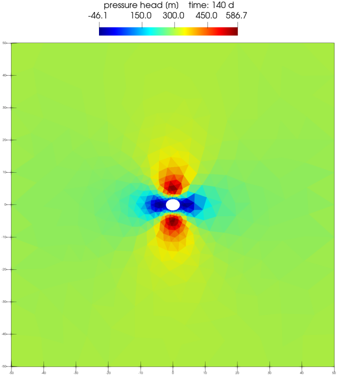
Rutqvist et al., 2009

Forward model in Flow123d

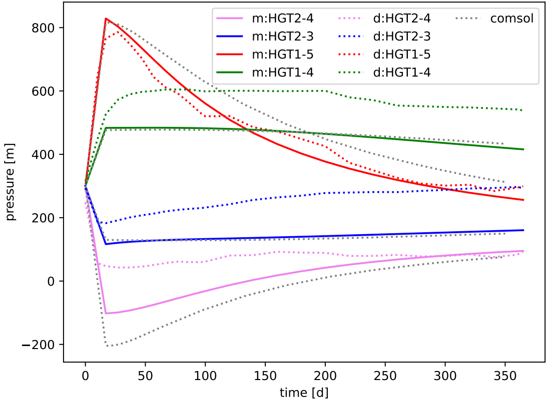
- 2d model
- linear poroelastic model
- homogeneous parameters derived from Ruquist et al., 2009

parameter	symbol	value	unit
h. conductivity	K	$6 \cdot 10^{-15}$	$\mathrm{m} \cdot \mathrm{s}^{-1}$
storativity	S	$2.8 \cdot 10^{-8}$	m^{-1}
initial pressure head	h_0	300	m
Biot's coef.	α	0.2	–
Young modulus	E	$6 \cdot 10^{10}$	Pa
Poisson coef.	ν	0.2	–

Results with fixed parameters



pressure head around the tunnel, day 140



simulation vs. experiment

Bayesian Inversion

Assumptions:

- forward model $G : \mathrm{R}^n \to \mathrm{R}^m$
- data $\boldsymbol{y} = G(\boldsymbol{u}) + \eta$, $\eta \sim \Phi$
- prior distribution $\pi_0(\boldsymbol{u})$, noise distribution Φ

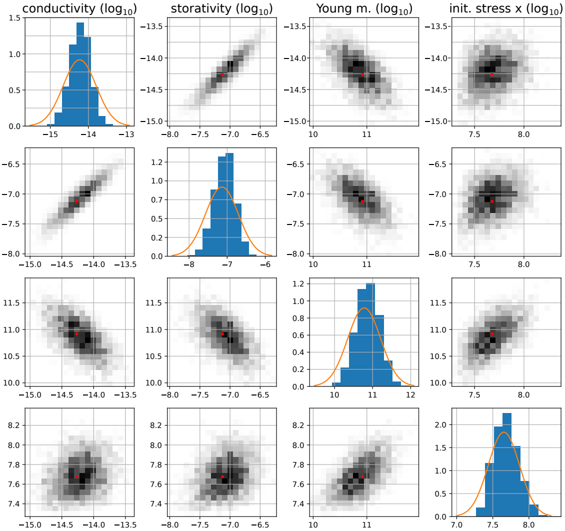
Bayes theorem:

$$\pi(\boldsymbol{u}|\boldsymbol{y}) = \frac{\pi(\boldsymbol{y}|\boldsymbol{u})\pi_0(\boldsymbol{u})}{\int \pi(\boldsymbol{y}|\boldsymbol{u})\pi_0(\boldsymbol{u})\,\mathrm{d}\boldsymbol{u}} \approx f(\boldsymbol{u}) := \Phi\left(\boldsymbol{y} - G(\boldsymbol{u})\right)\pi_0(\boldsymbol{u})$$

Metropolis-Hastings algorithm:

1. sample $\boldsymbol{v} \sim q(\boldsymbol{v}|\boldsymbol{u}^k)$ from *proposal density*, e.g. $N(\boldsymbol{u}^k, \sigma^2)$
2. $$\alpha = \min\left\{1, \frac{q(u^k|v)f(v)}{q(v|u^k)f(u^k)}\right\}$$
3. With probability α, accept: $\boldsymbol{u}^{k+1} := v$.

Bayesian inversion for TSX



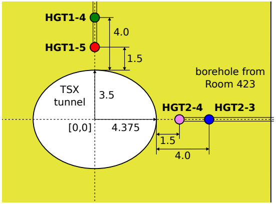
MH
992/5368

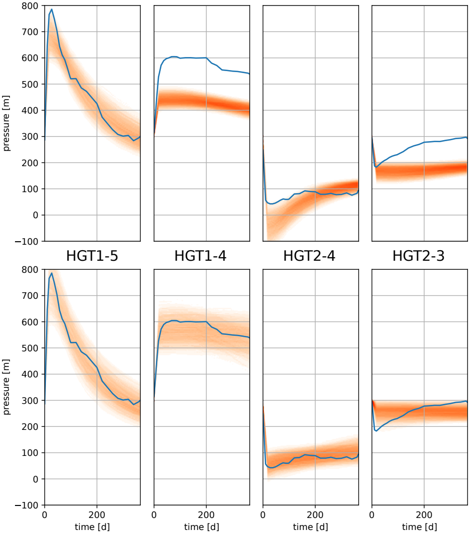
DAMH-SMU
3942/175

DAMH
3968/230

(14676 × direct model, 20 parallel chains)

Boreholes Comparison: all × one-by-one

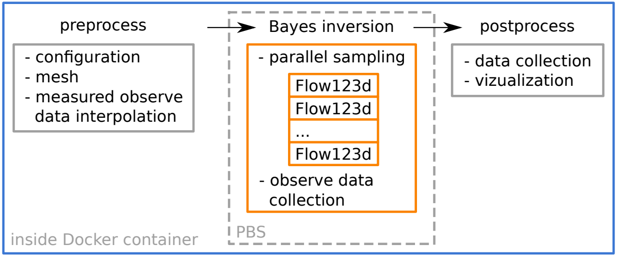




MPI + containers

Docker image

- `https://hub.docker.com/r/flow123d/ci-gnu/tags`
- includes Flow123d + GMSH + mpi4py + numpy + ...
- goal: can be run anywhere
  - locally – user's PC (CPUs/threading)
  - cluster – Metacentrum (CZ) – `singularity`, PBS



Conclusions

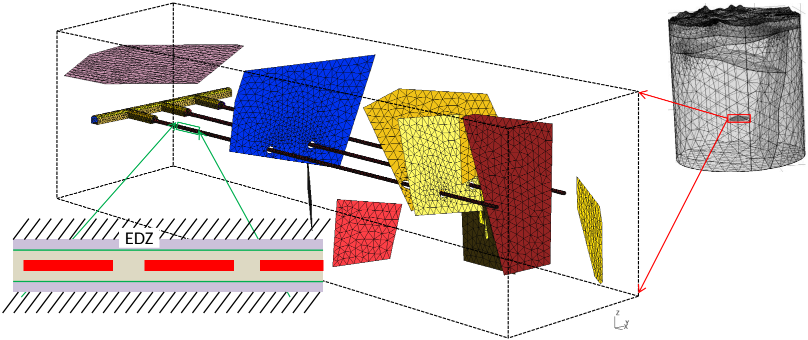
- demonstration of Bayesian inversion usage on a realistic model application
- significant speedup by surrogate models (see S. Bérešová)
- parallelism necessary, achieved by multiple Markov chains

Outlook:

- model of trapped air in the packer
- non-linear conductivity
- influence of plastic changes at tunnel wall
- heterogeneity / anisotropy

Outline

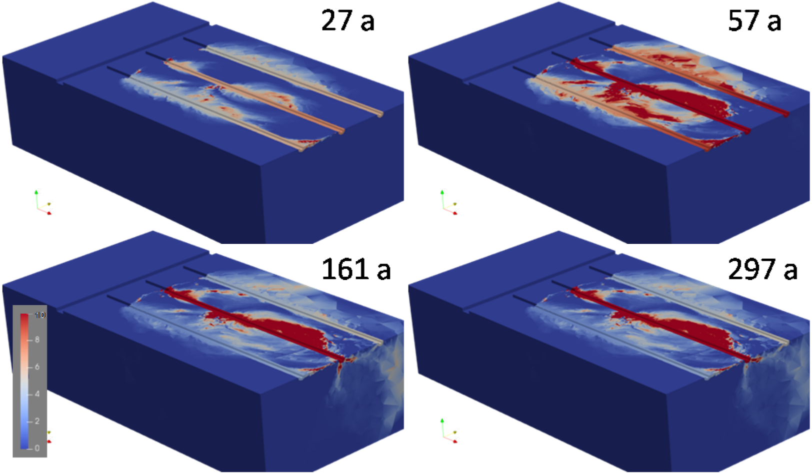
EDZ conceptual model



- Darcy flow driven by boundary pressure from a regional model
- source of contaminant $S(t)$, given on a single container position
- indicator given as maximal boundary concentration:

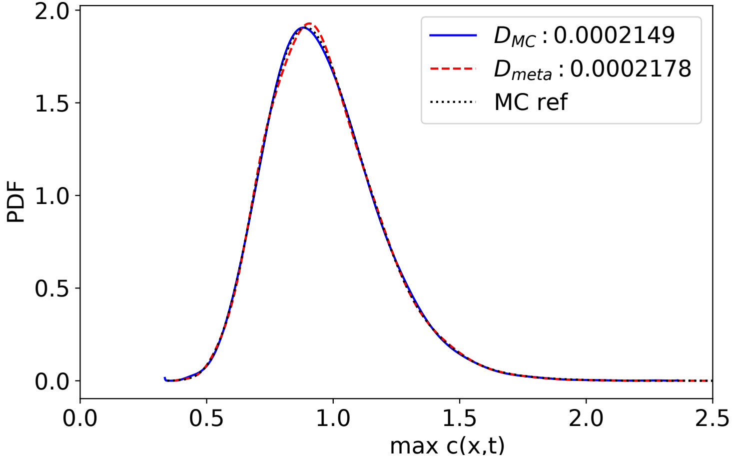
$$\max_{t<T} \max_{x\in\partial\Omega} c(x,t)$$

Results



MLMC + MEM PDF estimation

simplified 2D transport



Conclusions

- HM model for continuum + fractures
- key nonlinear effects included
- preliminary homogenization of fractures
- inversion with uncertainties
- uncertainty propagation

Outlook

- application of non-linear effects on fractures
- better forward models in Bayes
- technical completion of MLMC and 3D transport
- 3D homogenization